Position & Source: Position vector $\vec{r}$, source vector $\vec{r}'$, separation vector $\overrightarrow{\Delta r} = \vec{r} - \vec{r}'$

Fundamental Theorems of Vector Calculus:

$$\int_{\vec{a}}^{\vec{b}} \nabla f \cdot \overrightarrow{dl} = f(\vec{b}) - f(\vec{a}) \qquad \int \nabla \cdot \vec{A}\, d\tau = \oint \vec{A} \cdot \overrightarrow{da} \qquad \int (\nabla \times \vec{A}) \cdot \overrightarrow{da} = \oint \vec{A} \cdot \overrightarrow{dl}$$

Cartesian Coordinates: $\quad \overrightarrow{dl} = dx\hat{x} + dy\hat{y} + dz\hat{z} \qquad d\tau = dx\, dy\, dz$

$$\nabla f = \frac{\partial f}{\partial x}\hat{x} + \frac{\partial f}{\partial y}\hat{y} + \frac{\partial f}{\partial z}\hat{z} \qquad \nabla \cdot \vec{A} = \frac{\partial A_x}{\partial x} + \frac{\partial A_y}{\partial y} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \left(\frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} \right)\hat{x} + \left(\frac{\partial A_x}{\partial z} - \frac{\partial A_z}{\partial x} \right)\hat{y} + \left(\frac{\partial A_y}{\partial x} - \frac{\partial A_x}{\partial y} \right)\hat{z} \qquad \nabla^2 f = \frac{\partial^2 f}{\partial x^2} + \frac{\partial^2 f}{\partial y^2} + \frac{\partial^2 f}{\partial z^2}$$

Spherical Coordinates: $\quad x = r\sin\theta\cos\phi,\ y = r\sin\theta\sin\phi,\ z = r\cos\theta$

$$\overrightarrow{dl} = dr\hat{r} + r\,d\theta\hat{\theta} + r\sin\theta\,d\phi\hat{\phi} \qquad d\tau = r^2\sin\theta\,dr\,d\theta\,d\phi$$

$$\nabla f = \frac{\partial f}{\partial r}\hat{r} + \frac{1}{r}\frac{\partial f}{\partial \theta}\hat{\theta} + \frac{1}{r\sin\theta}\frac{\partial f}{\partial \phi}\hat{\phi} \qquad \nabla \cdot \vec{A} = \frac{1}{r^2}\frac{\partial (r^2 A_r)}{\partial r} + \frac{1}{r\sin\theta}\frac{\partial (\sin\theta\, A_\theta)}{\partial \theta} + \frac{1}{r\sin\theta}\frac{\partial A_\phi}{\partial \phi}$$

$$\nabla \times \vec{A} = \frac{1}{r\sin\theta}\left(\frac{\partial (\sin\theta\, A_\phi)}{\partial \theta} - \frac{\partial A_\theta}{\partial \phi} \right)\hat{r} + \frac{1}{r}\left(\frac{1}{\sin\theta}\frac{\partial A_r}{\partial \phi} - \frac{\partial (r\, A_\phi)}{\partial r} \right)\hat{\theta} + \frac{1}{r}\left(\frac{\partial (r\, A_\theta)}{\partial r} - \frac{\partial A_r}{\partial \theta} \right)\hat{\phi}$$

$$\nabla^2 f = \frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial f}{\partial r} \right) + \frac{1}{r^2\sin\theta}\frac{\partial}{\partial \theta}\left(\sin\theta\frac{\partial f}{\partial \theta} \right) + \frac{1}{r^2\sin^2\theta}\frac{\partial^2 f}{\partial \phi^2}$$

Cylindrical Coordinates: $\quad x = s\cos\phi,\ y = s\sin\phi,\ z = z$

$$\overrightarrow{dl} = ds\hat{s} + s\,d\phi\hat{\phi} + dz\hat{z} \qquad d\tau = s\,ds\,d\phi\,dz$$

$$\nabla f = \frac{\partial f}{\partial s}\hat{s} + \frac{1}{s}\frac{\partial f}{\partial \phi}\hat{\phi} + \frac{\partial f}{\partial z}\hat{z} \qquad \nabla \cdot \vec{A} = \frac{1}{s}\frac{\partial (s\, A_s)}{\partial s} + \frac{1}{s}\frac{\partial A_\phi}{\partial \phi} + \frac{\partial A_z}{\partial z}$$

$$\nabla \times \vec{A} = \left(\frac{1}{s}\frac{\partial A_z}{\partial \phi} - \frac{\partial A_\phi}{\partial z} \right)\hat{s} + \left(\frac{\partial A_s}{\partial z} - \frac{\partial A_z}{\partial s} \right)\hat{\phi} + \frac{1}{s}\left(\frac{\partial (s\, A_\phi)}{\partial s} - \frac{\partial A_s}{\partial \phi} \right)\hat{z} \qquad \nabla^2 f = \frac{1}{s}\frac{\partial}{\partial s}\left(s\frac{\partial f}{\partial s} \right) + \frac{1}{s^2}\frac{\partial^2 f}{\partial \phi^2} + \frac{\partial^2 f}{\partial z^2}$$

Dirac Delta Function: $\int \delta^3(\vec{r} - \vec{a})\,d\tau = 1$ if $\vec{a}$ contained in volume, $\delta^3(\overrightarrow{\Delta r}) = \frac{1}{4\pi}\nabla \cdot \left(\frac{\widehat{\Delta r}}{\Delta r^2} \right)$

Irrotational Functions: $\quad \nabla \times \vec{A} = 0 \qquad \vec{A} = \nabla f \qquad \oint \vec{A} \cdot \overrightarrow{dl} = 0$
Solenoidal Functions: $\quad \nabla \cdot \vec{F} = 0 \qquad \vec{F} = \nabla \times \vec{A} \qquad \oint \vec{F} \cdot \overrightarrow{da} = 0$

Electric Field: $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\overrightarrow{r\prime})}{\Delta r^2}\widehat{\Delta r}\, d\tau'$, $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma(\overrightarrow{r\prime})}{\Delta r^2}\widehat{\Delta r}\, da'$, $\vec{E}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda(\overrightarrow{r\prime})}{\Delta r^2}\widehat{\Delta r}\, dl'$

Gauss's Law: $\oint \vec{E}\cdot \overrightarrow{da} = Q_{enc}/\varepsilon_0$, $\nabla\cdot\vec{E} = \rho/\varepsilon_0$, $\quad \oint \vec{D}\cdot\overrightarrow{da} = Q_{f\_enc}$, $\nabla\cdot\vec{D} = \rho_{free}$

Electric Potential: $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\rho(\overrightarrow{r\prime})}{\Delta r}d\tau'$, $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\sigma(\overrightarrow{r\prime})}{\Delta r}da'$, $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\int \frac{\lambda(\overrightarrow{r\prime})}{\Delta r}dl'$
$\vec{E} = -\nabla V$, $\quad V(\vec{b}) - V(\vec{a}) = -\int_{\vec{a}}^{\vec{b}}\vec{E}\cdot\overrightarrow{dl}$, $\quad \nabla^2 V = -\rho/\varepsilon_0$

Laplace's Equation: $\nabla^2 V = 0$ if $\rho = 0$

Separation of Variables: $\frac{d^2 X}{dx^2} = C_1 X, \frac{d^2 Y}{dy^2} = C_2 Y, \frac{d^2 Z}{dz^2} = C_3 Z$, $C_1 + C_2 + C_3 = 0$, $V = X(x)Y(y)Z(z)$

Separation of Variables (Spherical): $V(r,\theta) = \sum_0^\infty \left(A_l r^l + \frac{B_l}{r^{l+1}}\right)P_l(cos\theta)$
$P_0(cos\,\theta) = 1, P_1(cos\,\theta) = cos\,\theta, P_2(cos\,\theta) = \frac{3cos^2\theta - 1}{2}$

Multipole Expansion: $V(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\sum_0^\infty \frac{1}{r^{n+1}}\int (r')^n P_n(cos\alpha)\rho(\vec{r}')d\tau'$

Dipoles: $\vec{p} = \int \vec{r}'\rho(\vec{r}')d\tau'$, $\vec{p} = \int \vec{r}'\sigma(\vec{r}')da'$, $\vec{p} = \int \vec{r}'\lambda(\vec{r}')dl'$
$V_{dip}(\vec{r}) = \frac{1}{4\pi\varepsilon_0}\frac{\vec{p}\cdot\hat{r}}{r^2}$, $\quad \vec{\tau} = \vec{p}\times\vec{E}$, $\quad \vec{F} = (\vec{p}\cdot\nabla)\vec{E}$, $\quad \vec{P} = \vec{p}/volume$

Boundary Conditions: $\Delta V = 0$, $\Delta\vec{E} = \frac{\sigma}{\varepsilon_0}\hat{n}$, $\quad \Delta D_\perp = \sigma_f$, $\quad \Delta\vec{D}_{||} = \Delta\vec{P}_{||}$

Work and Energy: $W = Q\Delta V$, $\quad W_{electrostatic} = \frac{\varepsilon_0}{2}\int E^2 d\tau$, $\quad W_{elec+polarization} = \frac{1}{2}\int \vec{E}\cdot\vec{D}\,d\tau$

Dielectrics: $\vec{D} = \varepsilon_0\vec{E} + \vec{P}$, $\quad \sigma_b = \vec{P}\cdot\hat{n}$, $\rho_b = -\nabla\cdot\vec{P}$

Linear Dielectrics: $\vec{P} = \varepsilon_0\chi_e\vec{E}$, $\quad \vec{D} = \varepsilon\vec{E} = \varepsilon_r\varepsilon_0\vec{E} = (1 + \chi_e)\varepsilon_0\vec{E}$